LIBERTY PAPER SET

STD. 10 : Mathematics (Basic) [N-018(E)]

Full Solution

Time : 3 Hours **ASSIGNTMENT PAPER 5**

Section-A

1. (A) Infinite **2.** (B) $\sqrt{x^2 + y^2}$ **3.** (A) $a_n = a + (n - 1)d$ **4.** (B) $D = b^2 - 4ac$ **5.** (B) $\tan\theta$ **6.** (B) 2 **7.** 5 **8.** downward open parabola **9.** – 1.5% **10.** 1 **11.** One **12.** 2.45 **13.** True **14.** True **15.** True **16.** True **17.** 14 **18.** 60° **19.** $\frac{1}{4}$ **20.** 40 **21.** (c) $\frac{1}{3}\pi r^2h$ **22.** (a) πr^2h **23.** (b) $\frac{\pi r^2\theta}{360}$ **24.** (a) $\frac{\pi r\theta}{180}$

Section-B

25. Suppose the bionomial polynomial $ax^2 + bx + c$ of zeroes is α and β.

$\therefore \alpha + \beta = -3$ and $\alpha\beta = 2$

$\therefore -\frac{b}{a} = \frac{-3}{1}$ and $\frac{c}{a} = \frac{2}{1}$

$\therefore a = 1, b = 3, c = 2$

So, one bionomial polynomial which fits the given conditions is $x^2 + 3x + 2$. You can check that any other bionomial polynomial that fits these conditions will be of the form $k(x^2 + 3x + 2)$, where k is real.

26. $p(x) = 2x^2 + 6x + 3$

a = 2, b = 6, c = 3

Sum of zeroes $= \frac{-b}{a} = \frac{-6}{2} = -3$

Product of zeroes $= \frac{c}{a} = \frac{3}{2}$

27. $2x^2 - 6x + 3 = 0$

$\therefore a = 2, b = -6$ and $c = 3$

$\therefore b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = 12$

Here $b^2 - 4ac > 0$, therefore, there are distinct real roots exist for given equation.

Now, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \frac{-(-6) \pm \sqrt{12}}{2 \times 2}$

$\therefore x = \frac{6 \pm 2\sqrt{3}}{4}$

$\therefore x = \frac{3 \pm \sqrt{3}}{2}$

Therefore, roots of given equation : $\frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$

28. $a = 5,\ d = 3,\ a_n = 50,\ n =$ ____

$a_n = a + (n - 1)d$

$\therefore\ 50 = 5 + (n - 1)3$

$\therefore\ 50 - 5 = (n - 1)3$

$\therefore\ 45 = (n - 1)3$

$\therefore\ n - 1 = \dfrac{45}{3}$

$\therefore\ n - 1 = 15$

$\therefore\ n = 16$

29. $a = 10,\ d = 7 - 10 = -3,\ n = 30,\ a_{30} =$ ______

$a_n = a + (n - 1)d$

$\therefore\ a_{30} = 10 + (30 - 1)(-3) = 10 + (29)(-3) = 10 - 87 = -77$

$\therefore\ a_{30} = -77$

30. Let the given points be A $(-5, 7)$ & B $(-1, 3)$

$$\therefore\ AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-5+1)^2 + (7-3)^2}$$

$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

Thus, the distance between the given points is $4\sqrt{2}$.

31. Let, the given points be A $(5, -2)$, B $(6, 4)$ & C $(7, -2)$.

$$AB = \sqrt{(5-6)^2 + (-2-4)^2} = \sqrt{1+36} = \sqrt{37}$$

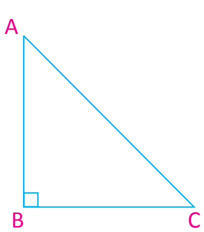
$$BC = \sqrt{(6-7)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37}$$

$$AC = \sqrt{(5-7)^2 + (-2+2)^2} = \sqrt{4+0} = \sqrt{4} = 2$$

Here, since PQ = QR in Δ PQR is an isosceles triangle,

thus, the points $(5, -2)$, $(6, 4)$ and $(7, -2)$ are the vertices of the isosceles triangle.

32.



$sin\ A = \dfrac{3}{4}$

In right angled Δ ABC, $\angle B = 90°$

$$sin\ A\ \frac{BC}{AC} = \frac{3}{4}$$

$$\therefore\ \frac{BC}{3} = \frac{AC}{4}\ \ k,\ k = \text{Positive Real Number}$$

$\therefore\ BC = 3k,\ AC = 4k$

According to pythagoras

$AB^2 = AC^2 - BC^2$

$\therefore \quad AB^2 = (4k)^2 - (3k)^2$

$\therefore \quad AB^2 = 16k^2 - 9k^2$

$\therefore \quad AB^2 = 7k^2$

$\therefore \quad AB = \sqrt{7}\,k$

$\therefore \quad cos\ A = \frac{AB}{AC} = \frac{\sqrt{7}\,k}{4k} = \frac{\sqrt{7}}{4}$

$tan\ A = \frac{sin\ A}{cos\ A} = \frac{\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{3}{\sqrt{7}}$

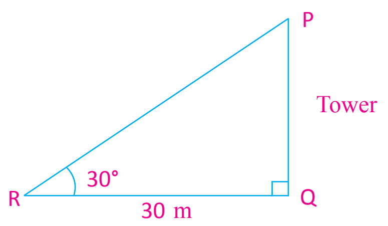
33. $2tan^2 45° + cos^2 30° - sin^2 60°$

$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2 + \frac{3}{4} - \frac{3}{4}$

$= 2$

34.



Here, PQ represents the tower, P is the top of the tower and point R is the point of observation.

In Δ PQR, ∠Q = 90°, ∠R = 30° and QR = 30 m.

$\therefore \quad tan\ R = \frac{PQ}{QR}$

$\therefore \quad tan\ 30° = \frac{PQ}{30}$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{PQ}{30}$

$\therefore \quad PQ = \frac{30}{\sqrt{3}} = 10\sqrt{3}$ m

Hence, the height of the tower is $10\sqrt{3}$ m.

35. Suppose, the side length of the cube be x.

$\therefore$ Volume of cube $= x^3$

$\therefore\ 64 = x^3$

$\therefore\ x = 4$ cm

$l = 2x = 2 \times 4 = 8$ cm, $b = x = 4$ cm and

$h = x = 4$ cm

$\therefore$ Area of rectangle formed $= 2\ (lb + bh + hl)$

$= 2\ (8 \times 4 + 4 \times 4 + 4 \times 8)$

$= 2\ (32 + 16 + 32)$

$= 2(80)$

$= 160$ cm^2

36. $r = h = 7$ cm

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7^2 \times 7$$

$$= 22 \times 49$$

$$= 1078 \text{ cm}^3$$

37. We have,

$$\text{Mode Z} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 40 + \left[\frac{7-3}{2(7)-3-6}\right] \times 15$$

$$= 40 + \left[\frac{4}{14-9}\right] \times 15$$

$$= 40 + \left(\frac{4}{5}\right) \times 15$$

$$= 40 + \left(\frac{4 \times 5 \times 3}{5}\right)$$

$$= 40 + (4 \times 3)$$

$$= 40 + 12$$

$$Z = 52$$

38. $\sqrt{2}\,x + \sqrt{3}\,y = 0$ ...(1)

$\therefore x = \dfrac{-\sqrt{3}\,y}{\sqrt{2}}$ ...(2)

$\sqrt{3}\,x - \sqrt{8}\,y = 0$ ...(3)

Put value of equation (2) in equation (3),

$$\sqrt{3}\,x - \sqrt{8}\,y = 0$$

$$\therefore \sqrt{3}\left(\frac{-\sqrt{3}\,y}{\sqrt{2}}\right) - \sqrt{8}\,y = 0$$

$$\therefore \frac{-3y}{\sqrt{2}} - \sqrt{8}\,y = 0$$

$$\therefore \frac{-3y - 4y}{\sqrt{2}} = 0$$

$$\therefore -7y = 0$$

$$\therefore y = 0$$

Put $y = 0$ in equation (2)

$$x = \frac{-\sqrt{3}\,y}{\sqrt{2}}$$

$$\therefore x = \frac{-\sqrt{3}(0)}{\sqrt{2}}$$

$$\therefore x = 0$$

Therefore, the solution is : $x = 0$, $y = 0$

39. Let us assume that,

Bhavin's present age = x

Vrutik's present age = y

Five years ago,

Bhavin = $x - 5$

Vrutik = $y - 5$

$\therefore$ As per condition $(x - 5) = 3(y - 5)$

$\therefore$ $x - 5 = 3y - 15$

$\therefore$ $x - 3y = -15 + 5$

$\therefore$ $x - 3y = -10$ ...(1)

10 years from now,

Bhavin will be $x + 0$ & Vrutik will be $y + 10$

As per condition $(x + 10) = 2(y + 10)$

$\therefore$ $x + 10 = 2y + 20$

$\therefore$ $x - 2y = 20 - 10$

$\therefore$ $x - 2y = 10$ ...(2)

Subtract (2) from (1),

$x - 3y = -10$

$x - 2y = 10$

$- \quad + \quad +$

$\therefore$ $-y + 20 = 0$

$\therefore$ $y = -20$

$y = 20$

Put $y = 20$ in eqn (1),

$x - 3y = -10$

$\therefore$ $x - 3(20) = -10$

$\therefore$ $x - 60 = -10$

$\therefore$ $x = -10 + 60$

$\therefore$ $x = 50$

Bhavin's present age = 50 years, Vrutik's present age = 20 years.

40. $a_{12} = 37$, $d = 3$, a = ______, S_{12} = ______

Now, $a_{12} = 37$

$\therefore$ $a + 11d = 37$

$\therefore$ $a + 11(3) = 37$

$\therefore$ $a + 33 = 37$

$\therefore$ $a = 37 - 33$

$\therefore$ $a = 4$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore$ $S_{12} = \frac{12}{2}[2(4) + (12 - 1)(3)]$

$= 6[8 + 33]$

$= 6 \times 41$

$\therefore$ $S_{12} = 246$

41. Here, A (6, 1), B (8, 2), C (9, 4) and D (P, 3) are the vertices of the parallelogram ABCD.

$\therefore$ Co-ordinates from the midpoint of the diagonal AC = Co-ordinates from the midpoint of the diagonal BD

$$\therefore \left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+P}{2}, \frac{2+3}{2}\right)$$

$$\therefore \left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+P}{2}, \frac{5}{2}\right)$$

$$\therefore \frac{15}{2} = \frac{8+P}{2}$$

$\therefore$ 15 = 8 + P

$\therefore$ P = 7

42. Let the point P on x-axis be $(x, 0)$ which is at equal distances from points A (2, – 5) and B (– 2, 9).

$\therefore$ PA = PB

$\therefore$ $PA^2 = PB^2$

$\therefore (x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$

$\therefore x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$

$\therefore -4x - 4x = 4 + 81 - 4 - 25$

$\therefore -8x = 56$

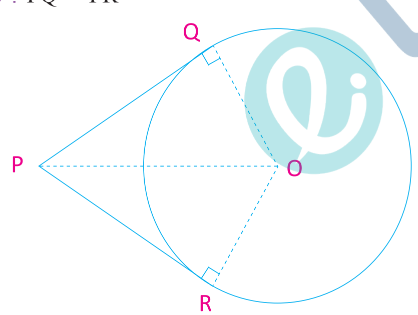
$\therefore x = -7$

Hence, the required point on the x-axis is (– 7, 0).

43. Given : A circle with centre O, a point P lying outside the circle with two tangents PQ, PR on the circle from P.

To prove : PQ = PR

Figure :



Proof : Join OP, OQ and OR. Then ∠OQP and ∠ORP are right angles because these are angles between the radii and tangents and according to theorem 10.1 they are right angles.

Now, in right triangles OQP and ORP,

OQ = OR (Radii of the same circle)

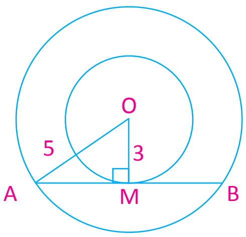
OP = OP (Common)

∠OQP = ∠ORP (Right angle)

Therefore, Δ OQP ≅ Δ ORP (RHS)

This gives, PQ = PR (CPCT)

44.



Here, chord AB of ⊙ (0, 5) touches. (0, 3) at point M.

Therefore, OM ⊥ AB and M is the midpoint of AB.

In Δ OMA; ∠OMA = 90°

∴ $AM^2 + OM^2 = OA^2$ (Pythagoras Theorem)

∴ $AM^2 + (3)^2 = (5)^2$

∴ $AM^2 + 9 = 25$

∴ $AM^2 = 25 - 9$

∴ $AM^2 = 16$

∴ AM = 4

But, AB = 2AM = 2 × 4

∴ AB = 8

Hence, the length of chord AB is 8 cm.

45.

Class Interval	No. of families (f_i)	x_i	u_i	f_iu_i
10 – 25	2	17.5	– 2	– 4
25 – 40	3	32.5	– 1	– 3
40 – 55	7	47.5 = a	0	0
55 – 70	6	62.5	1	6
70 – 85	6	77.5	2	12
85 – 100	6	92.5	3	18
Total	30			29

Mean $\bar{x} = a + \frac{\sum fiui}{\sum fi} \times h$

∴ $\bar{x} = 47.5 + \frac{29 \times 15}{30}$

∴ $\bar{x} = 47.5 + 14.5$

∴ $\bar{x} = 62$

46. Total number of letters = 5 + 8 + 1 = 14

(i) Suppose event A drawn letter is red.

∴ $P(A) = \frac{\text{Number of red letter}}{\text{Total number of letters}}$

∴ $P(A) = \frac{5}{14}$

(ii) Suppose event B drawn letter is white.

$$\therefore \ P(B) = \frac{\text{Number of white letter}}{\text{Total number of letters}}$$

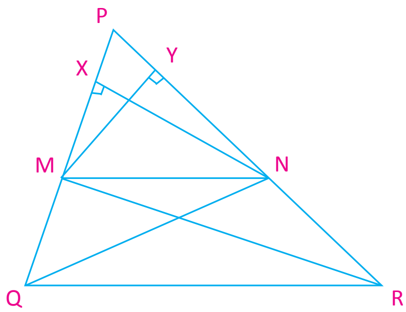
$$\therefore \ P(B) = \frac{8}{14} = \frac{4}{7}$$

(iii) Suppose event C drawn letter is not green.

$$\therefore \ P(C) = \frac{\text{Number of is not green letter}}{\text{Total number of letters}}$$

$$\therefore \ P(C) = \frac{13}{14}$$

47.



Given : In Δ PQR, a line parallel to side QR intersects PQ and PR at M and N respectively.

To prove : $\frac{PM}{MQ} = \frac{PN}{NR}$

Proof : Join QN and RM and also draw

MY ⊥ PR and NX ⊥ PQ

Then, PMN = $\frac{1}{2} \times PM \times NX$,

QMN = $\frac{1}{2} \times MQ \times NX$,

PMN = $\frac{1}{2} \times PN \times MY$ and

MNR = $\frac{1}{2} \times NR \times MY$

$$\therefore \ \frac{PMN}{QMN} = \frac{\frac{1}{2} \times PM \times NX}{\frac{1}{2} \times MQ \times NX} = \frac{PM}{MQ} \qquad ...(1)$$

$$\therefore \ \frac{PMN}{MNR} = \frac{\frac{1}{2} \times PN \times MY}{\frac{1}{2} \times NR \times MY} = \frac{PN}{NR} \qquad ...(2)$$

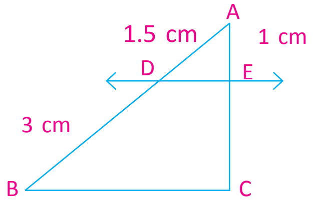
Now, Δ QMN and Δ MNR are triangles on the same base MN and between the parallel QR and MN.

$\therefore$ QMN = MNR ...(3)

Hence from eq^n, (1), (2) and (3)

$$\frac{PM}{MQ} = \frac{PN}{NR}$$

48.

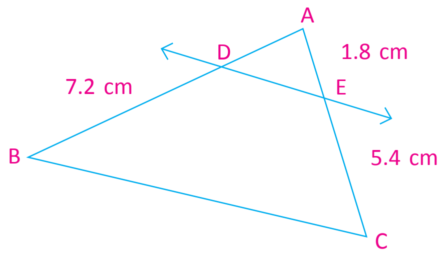


$$\frac{AD}{DB} = \frac{AE}{EC} \qquad \text{(Theorem : 6.1)}$$

$$\therefore \frac{1.5}{3} = \frac{1}{EC}$$

$\therefore$ EC = 2 cm

(ii)



$$\frac{AD}{DB} = \frac{AE}{EC} \qquad \text{(Theorem : 6.1)}$$

$$\therefore \frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$\therefore AD = \frac{1.8 \times 7.2}{5.4}$$

$\therefore$ AD = 2.4 cm

49. Let one of the odd positive integer be x

then the other odd positive integer is $x + 2$

their sum of squares $= x^2 + (x + 2)^2$

$= x^2 + x^2 + 4x + 4$

$= 2x^2 + 4x + 4$

Given that their sum of squares = 290

$2x^2 + 4x + 4 = 290$

$2x^2 + 4x = 290 - 4 = 286$

$2x^2 + 4x - 286 = 0$

$2(x^2 + 2x - 143) = 0$

$x^2 + 2x - 143 = 0$

$x^2 + 13x - 11x - 143 = 0$

$x(x + 13) - 11(x + 13) = 0$

$(x - 11)(x + 13) = 0$

$(x - 11) = 0$, OR $(x + 13) = 0$

Therefore, $x = 11$ or -13

We always take positive value of x

So, $x = 11$ and $(x + 2) = 11 + 2 = 13$

Therefore, the odd positive integers are 11 and 13.

50. Here, $a_3 = 5$

$\therefore\ a + 2d = 5$ ...(1)

$a_7 = 9$

$\therefore\ a + 6d = 9$ ...(2)

Subtract equation (2) by (1),

$(a + 2d) - (a + 6d) = 5 - 9$

$\therefore\ a + 2d - a - 6d = -4$

$\therefore\ -4d = -4$

$\therefore\ d = 1$

Put $d = 1$ in equation (1),

$a + 2d = 5$

$\therefore\ a + 2(1) = 5$

$\therefore\ a + 2 = 5$

$\therefore\ a = 3$

$\therefore\ a_1 = a = 3$

$a_2 = a + d = 3 + 1 = 4$

$a_3 = a + 2d = 3 + 2(1) = 3 + 2 = 5$

$a_4 = a + 3d = 3 + 3(1) = 3 + 3 = 6$

Hence, the required AP is 3, 4, 5, 6, 7, ...

51. Here, maximum class frequency is 23 which belong to class interval 35-45.

$\therefore\ l$ = lower limit of modal class = 35

h = class size = 10

f_1 = frequency of modal class = 23

f_0 = frequency of class prceeding the modal class = 21

f_2 = frequency of class succeeding the modal class = 14

Mode, $Z = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$\therefore\ Z = 35 + \left(\dfrac{23-21}{2(23)-21-14}\right) \times 10$

$\therefore\ Z = 35 + \dfrac{2 \times 10}{11}$

$\therefore\ Z = 35 + 1.82$

$\therefore\ Z = 36.82$ (Approx)

52.

Class intervals	Frequency	Cumulative frequency
0 – 100	2	2
100 – 200	5	7
200 – 300	x	$7 + x$
300 – 400	12	$19 + x$
400 – 500	17	$36 + x$
500 – 600	20	$56 + x$
600 – 700	y	$56 + x + y$
700 – 800	9	$65 + x + y$
800 – 900	7	$72 + x + y$
900 – 1000	4	$76 + x + y$

It is given that $n = 100$

$$\frac{n}{2} = \frac{100}{2} = 50$$

$\therefore\ 76 + x + y = 100$

$\therefore\ x + y = 24$

The median is 525, which lies in the class 500 – 600.

$l = 500$

$cf = 36 + x$

$f = 20$

$h = 100$

$$\text{Median M} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\therefore\ 525 = 500 + \left(\frac{50 - 36 - x}{20}\right) \times 100$$

$\therefore\ 525 - 500 = (14 - x)5$

$$\therefore\ \frac{25}{5} = 14 - x$$

$\therefore\ 5 = 14 - x$

$\therefore\ x = 14 - 5$

$\therefore\ x = 9$

Now, $x + y = 24$

$\therefore\ 9 + y = 24$

$\therefore\ y = 15$

53. Here, the number of possible outcomes = 52

(i) There are 4 aces in a deck. Let E be the event ‘the card is an ace’.

The number of outcomes favourable to

E = 4

$$\text{Therefore, P(E)} = \frac{4}{52} = \frac{1}{13}$$

(ii) Let F be the event 'card drawn is not an ace'.

The number of possible outcomes = 48

(52 – 4 = 48)

Therefore, $P(F) = \frac{48}{52} = \frac{12}{13}$

(iii) Let G be the event 'the card is a red ace'.

There are 1 red ace in a deck.

$\therefore P(G) = \frac{1}{52}$

(iv) Let H be the event 'the card is a black ace'.

There are 1 black ace in a deck.

$\therefore P(H) = \frac{1}{52}$

54. (i) Total number of bulbs = 20

Total number of defective bulbs = 4

$\therefore$ Total number of non-defective bulbs = 20 – 4 = 16

Suppose event A is drawing a defective bulbs

$\therefore P(A) = \frac{\text{Number of defective bulbs}}{\text{Total number of bulbs}} = \frac{4}{20}$

$\therefore P(A) = \frac{1}{5}$

(ii) Now bulb in is not defective and is not replaced.

Therefore, total number of non-defective bulbs is 15 and total number of defective bulbs is 4. Hence, total bulbs 19(15 + 4).

$\therefore$ Remaining number of bulbs = 19

Suppose event B is drawing bulb is not defective.

$\therefore P(B) = \frac{\text{Number of non-defective bulbs}}{\text{Total number of bulbs}}$

$\therefore P(B) = \frac{15}{19}$